Leave blank

1. $$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2\cos x$$

(a) Find $\frac{d^3y}{dx^3}$ in terms of x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(3)

At $x = 0$, $y = 1$ and $\frac{dy}{dx} = 3$

(b) Find the value of $\frac{d^3y}{dx^3}$ at $x = 0$

(1)

(c) Express y as a series in ascending powers of x, up to and including the term in x^3.

(3)

Leave blank

2. (a) Sketch, on the same axes,

(i) $y = |2x - 3|$

(ii) $y = 4 - x^2$

(3)

(b) Find the set of values of x for which

$$4 - x^2 > |2x - 3|$$

(6)

Leave blank

3. $$f(x) = \ln(1 + \sin kx)$$

where k is a constant, $x \in \mathbb{R}$ and $-\frac{\pi}{2} < kx < \frac{3\pi}{2}$

(a) Find $f'(x)$

(2)

(b) Show that $f''(x) = \dfrac{-k^2}{1 + \sin kx}$

(3)

(c) Find the Maclaurin series of $f(x)$, in ascending powers of x, up to and including the term in x^3.

(4)

Leave blank

4. Find the general solution of the differential equation

$$x\frac{dy}{dx} + (1 + x\cot x)y = \sin x, \qquad 0 < x < \pi$$

giving your answer in the form $y = f(x)$.

(9)

Leave blank

5\. (a) Express $\frac{2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$$

(4)

Leave blank

6. Solve the equation

$$z^5 = -16\sqrt{3} + 16\text{i}$$

giving your answers in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta < \pi$.

(8)

Leave blank

7. (a) Find the value of the constant λ for which $y = \lambda x\mathrm{e}^{2x}$ is a particular integral of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4y = 6\mathrm{e}^{2x}$$

(4)

(b) Hence, or otherwise, find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4y = 6\mathrm{e}^{2x}$$

(3)

Leave blank

8. A complex number z is represented by the point P on an Argand diagram.

(a) Given that $|z| = 1$, sketch the locus of P.

(1)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 7\text{i}}{z - 2\text{i}}$$

(b) Show that T maps $|z| = 1$ onto a circle in the w-plane.

(5)

(c) Show that this circle has its centre at $w = -5$ and find its radius.

(2)

P 4 1 8 0 4 A 0 2 2 2 8

Leave blank

9.

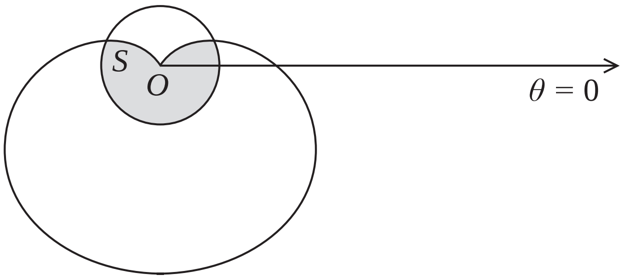

Figure 1

Figure 1 shows a sketch of the curves given by the polar equations

$$r = 1 \quad \text{and} \quad r = 2 - 2\sin\theta$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r < 1$ and for which $r < 2 - 2\sin\theta$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are rational numbers.

(8)